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Dynamic behavior of a social model for opinion formation

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The dynamic behavior of a social group influenced by both a strong leader and the mass media, which is modeled according to the social impact theory, is studied under two situations: (i) The strong leader changes his/her state of opinion periodically while the mass media are not considered. In this case, the leader is capable of driving the group between a dynamically ordered state with a weak leader-group coupling (high-frequency regime) and a dynamically disordered state where the group follows the opinion of the leader (low-frequency regime). (ii) The mass-media change periodically their message and have to compete with a strong leader that keeps his/her state of opinion unchanged. In this case, the mass media require an amplitude threshold in order to overcome the influence of the leader and drive the system into a dynamically disordered state. The dynamic behavior characteristic of the studied social opinion model shares many features of physical systems that are relevant in the fields of statistical mechanics and condensed matter.

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I. INTRODUCTION

The study of models describing far-from-equilibrium systems by means of statistical physics methods has become a subject of interdisciplinary interest, including physics, chemistry, biology, economics, social sciences [1–6], etc. Particularly, the study of the social behavior of groups of individuals has attracted the interest of many physicists. Among the various topics addressed in this field, it has been found that models for opinion formation in a society exhibit a rich variety of physicslike behavior, such as phase transitions and critical phenomena, stochastic resonance, chaos, bistability, etc. [7–18], for a short review see, e.g., Ref. [19]. A common feature of most models of opinion formation is the occurrence of abrupt (first-order-like) transitions between two different states of opinion [16–19]. These transitions may be driven by different control parameters such as the mass media, the noise, etc. In actual physical systems, first-order transitions are characterized by the presence of bistability and hysteretic effects. This scenario is essential for the observation of another interesting physical phenomenon: dynamic phase transitions (DPT's) [20–27]. DPT's may be observed when a physical system is forced by an oscillatory (external) drive. In this case, one has a symmetry breaking between a dynamically ordered state such that the system cannot follow the external oscillatory drive and a dynamically disordered state where the system becomes coupled to the drive. Of course, DPT's involve the competition of two characteristic time scales: the relaxation time of a stationary state of the physical system and the period of the external drive. DPT's have been observed, among others, in the Ising magnet [20–27], the Heisenberg model [28,29], the *XY* model [30], the Ziff-Gulari-Barshad (ZGB) model for the catalytic oxidation of CO [31,32], and very recently for the underpotential deposition of Ag on $A(111)$ [33]. Within this context, the aim of this paper is to study and characterize DPT's in a model for opinion formation based on the social impact theory of Latané [34], which is known to exhibit sharp first-order-like transitions due to the competition between a strong leader and the mass media [16–19].

II. THE MODEL OF OPINION FORMATION AND THE SOCIAL IMPACT THEORY

In order to formulate a model of opinion formation in the presence of a leader, we will follow the ideas proposed by Hołyst *et al.* [16–18] and take into account the three principles of the social impact theory developed by Latané [34] and its subsequent generalization performed in order to account for the dynamic evolution of the social groups [35]. It is worth mentioning that the dynamic generalization of the social impact theory has also been applied to the modeling of social interactions [36] and more recently as an approach to the teaching-learning process in a classroom context [37–39], where precursors of phase transitions similar to those observed in magnetic systems have been identified. Also, the same approach has been used to mimic the process of social learning via the Internet (chatting) [40]. In a related context, a model for the dynamics of social influence aimed to describe the coordination of individual economic decisions has also been formulated on the basis of the social impact theory [41].

According to Latané [34], when the interaction between individuals of a social group is considered, the impact (I) of an individual (the source) on another one (the target) depends on at least two factors (i) the "strength" or "intensity" of the source to the target, which is determined, for example, by the source's social and economic status, age, its credibility to persuade and become supported, prior relationships with or future power over the target, etc., and (ii) the "immediacy" that accounts for the social "distance" between the source and the target, which is determined, for example, by social, cultural, economic, and religious affinities, etc.

Since we are interested in cases where the number of individuals (N) may be large ($N>2$), a careful treatment of the situation becomes necessary. In fact, as pointed out by

Latané [34], if one considers the social impact of many sources on a single target, the psychosocial law (i.e., the second principle of Latané's theory) states that a marginal decreasing effect on I operates, and the impact is not simply proportional to N but one has $I \propto N^{\alpha}$, where $\alpha \leqslant 1$ is an exponent [19,40].

On the other hand, when a single source of impact acting on many targets is considered, the division of the social impact (i.e., the third principle of Latané's theory [34]) becomes relevant and one has $I \propto N^{-\beta}$, where β is also an exponent. This effect accounts for the fact that an individual, in the presence of many others, will feel a diminution of the impact as compared to the case that he/she would be alone [19,40].

The opinion of every individual $(j=1,2,\ldots,N)$ is accounted for by a two-state "spin" variable $\sigma_j = \pm 1$. The strength of the jth individual is denoted by S_j. Furthermore, the immediacy between the jth and the ith individuals is given by n_{ij}, such that in general one has $n_{ij} \neq n_{ji}$.

So, the social impact on the ith individual (I_i) is given by [16,17]

$$I_i = \sum_{j=1}^{N} S_j n_{ij} \sigma_j. \tag{1}$$

Equation (1) implies that the impact of all sources has the same statistical weight. So, in order to account for a marginal decreasing effect on the impact as stated by the second principle of Latané's theory, we define the impact acting on an individual of the group according to

$$I_i^G = (I_i/N) N^{\alpha} = I_i N^{\alpha-1}, \tag{2}$$

where $\alpha \leqslant 1$ is an exponent. Notice that for $\alpha = 1$ one has again the same statistical weight for all sources of impact, while the second principle becomes operative for $\alpha < 1$.

Now, let us introduce an external impact h acting uniformly on all N individuals and favoring one of the states of opinion. For example, this source may account for the influence of the mass media. Since this source impacts over many individuals, the third principle of Latané's theory has to be considered. So, the impact of the external source on the individuals is given by (I^E)

$$I^E = h/N^{\beta} = hN^{-\beta}, \tag{3}$$

where β is an exponent. Of course, there may be many situations where the division of the impact due to the mass media does not operate, e.g., when isolated agents read newspapers, watch TV, etc., without any contact with the social group. In that case one may consider a more generalized model, such that a fraction (say f) of the agents undergoes impact division since they are engaged in social activities, while for the remaining fraction of isolated agents $(1-f)$ the division of the impact does not apply. Performing this kind of study is beyond the scope of this paper.

Summing up, the total impact on the ith individual is given by the contribution of Eqs. (2) and (3), so one has

$$I_i^T = \left(\sum_{j=1}^{N} S_j n_{ij} \sigma_j \right) N^{\alpha-1} + hN^{-\beta}. \tag{4}$$

As pointed out by Latané [34], his original theory is essentially static: it provides laws to evaluate the impact, and the dynamic behavior of the targets of such an impact is not addressed. This shortcoming has already been overcome by the subsequent statistical formulation of the theory [35]. In our case, the impact influences the opinion of the individuals, which changes in time according to the following dynamic rule:

$$\sigma_j(t+1) = \begin{cases} +1 & \text{with probability } \mathbf{p}, \\ -1 & \text{with probability } 1-\mathbf{p}, \end{cases} \tag{5}$$

respectively, where $\mathbf{p}$ is given by

$$\mathbf{p} = \exp(I_i^T/T_s)/[\exp(-I_i^T/T_s) + \exp(I_i^T)/T_s]. \tag{6}$$

So, in terms of the language used for the statistical mechanics treatment of interacting spin systems, the proposed model consists of Ising spins with long-range coupling constants $(J_{ij} \equiv S_j n_{ij})$, which are random quenched and nonsymmetric $(J_{ij} \neq J_{ji})$ parameters. There is also a self-interaction term reflecting the inertia of a spin to become flipped and an external magnetic field acting uniformly on all spins. Furthermore, one "leader" spin has a large coupling constant and it is set always to spin up. In this way, Eq. (4) can be thought of as the "energy" of a single agent, while the "Hamiltonian" of the system would be the sum of all these energies. Furthermore, in Eqs. (5) and (6) one can recognize a heat-bath-like transition probability [42–44], where T_s is the so-called "social temperature." This temperature has already been introduced by various authors [16,17,37,39,40], and references therein, in order to evaluate transition rates in social systems, as in the case given by Eq. (6). The social temperature accounts for the noise (misunderstandings, lack of attention, etc.) in the communications among individuals in the social group. The social temperature has also been introduced by Grabowski *et al.* [45] in an Ising-based model for opinion formation. In this case it is assumed that T_s may be connected to the situation of the community, which may be described by the economic status of the people, employment, crime wave, etc.

The system under study is a cellular automata, so all individuals are updated simultaneously during each (discrete) time step (TS). The unity of time is just TS. Furthermore, the strength parameters of the interaction S_j are random variables uniformly distributed within the interval $0 < S_j \leqslant 2\langle S \rangle$, where $\langle S \rangle$ is the average strength of the individuals, which in the simulations is taken as $\langle S \rangle = 1$.

In order to introduce a strong leader we assume that one individual (say individual $j=1$) does not change his/her opinion $(\sigma_1 = 1)$ and his/her degree of impact on the society is greater than the average. The latter condition is simply achieved by taking $S_L \equiv S_1 \gg \langle S \rangle$.

For the sake of simplicity, the immediacy is taken as the inverse of the geometric distance between individuals placed in a ring $(n_{ij} = 1/d_{ij})$. Generalizations to more complex topologies are straightforward. On the other hand, the self-immediacy term (n_{jj}) describes the self-support of the individual that represents the social inertia preventing his/her change of opinion. In the simulations n_{jj} is taken at random and uniformly distributed according to $0 < n_{jj} < 2\langle S_S \rangle$, where $\langle S_S \rangle$ is the average value of the self-support, which is taken as $\langle S_S \rangle = 5$.

III. DYNAMIC PHASE TRANSITIONS

The archetypical model for the study of DPT's is the Ising magnet [42–44]. It is well known that in the absence of a magnetic field the Ising magnet undergoes a second-order phase transition between an ordered state with a nonvanishing spontaneous magnetization and a disordered state of zero magnetization. The transition occurs at a certain critical temperature T_c, in dimension $d=2$. By applying a magnetic field and for $T < T_c$, one observes sharp first-order transitions between states of different sign of the magnetization [42–44]. Of course, one also observes hysteretic effects and a relevant parameter is the relaxation time (τ_R) of a given state, which is determined just by measuring the time dependence of magnetization [$m(t)$] when the system starts from a state such that $m > 0$ and $T < T_c$, and subsequently a field $H < 0$ is applied. Then, by applying an oscillatory field of period τ_H, one can recognize two well-defined situations: (i) for $\tau_H \ll \tau_R$ the system can no longer follow the external drive, while (ii) for $\tau_H \gg \tau_R$ the system becomes coupled to the field. Symmetry breaking between these two states gives rise to a DPT at a certain critical period, which takes place between a dynamically ordered state ($\tau_H \ll \tau_R$) and a dynamically disordered one ($\tau_H \gg \tau_R$) [20–27].

Now, focusing our attention on the opinion formation model described in the previous section, one can consider at least two scenarios leading to the occurrence of DPT's, namely, (i) the leader changes his/her opinion while the mass media remain unchanged and (ii) the leader stands for his/her opinion but the mass media change their opinion. In both cases one can easily calculate the average response opinion (per individual) to the drive at time t, given by

$$\sigma(t) = \frac{1}{N}\sum_{i=1}^{N} \sigma_i(t). \quad (7)$$

One is often interested in the behavior of this response function for a long time $t \gg \tau_H$. So, in order to characterize the system, it is useful to define the dynamic order parameter Q given by

$$Q = \frac{1}{\tau_{Ex}} \oint \sigma(t) dt, \quad (8)$$

where τ_{Ex} is the period of the forcing drive.

IV. RESULTS AND DISCUSSION

According to the mean-field results of Hołyst *et al.* [16–18] and our previous Monte Carlo simulations [19,40], it is well established that the social model of opinion formation outlined in previous sections exhibits sharp first-order-like transitions between two states of opinion: a state where the opinion of the leader prevails and another one where the opinion of the mass media dominates the social group. While

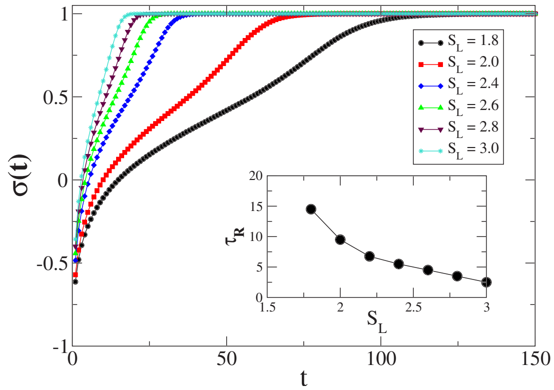

FIG. 1. (Color online) Plot of the averaged opinion of social group $\sigma(t)$ versus t. Results obtained by keeping the opinion of the leader fixed at $\sigma_L \equiv 1$ and starting with the remaining $(N-1)$ individuals in the opposite state of opinion. Results corresponding to different values of the strength of the leader S_L are shown. The inset shows the dependence of the relaxation time (τ_R), defined as the time required to achieve a neutral opinion, versus S_L. Data obtained for $N=2048$ individuals, by averaging over 500 different samples. More details in the text.

interesting finite-size effects, which are relevant in order to understand the behavior of small communities, are observed for relatively small samples ($L < 512$), the existence of sharp transitions is characteristic of large samples and particularly within the low social temperature range [19,40]. Abrupt transitions are also observed for a wide range of values of the parameters controlling the weakening of the social impact, according to the second and third principles of the social impact theory [19,40], namely, α and β, respectively. In view of these findings, in the present paper we restricted ourselves to the study of a typical case by choosing $\alpha=0.85$ and $\beta=0.5$. In view of preliminary results [40], the same qualitative behavior is expected to hold for other values of the exponents, except of course when the division of the impact is maximum and the transition of the opinion formation model is no longer of first order but it becomes of second order [19]. Furthermore, we focus our attention on the low social temperature regime of the model by taking $T_s=0.001$. All these parameters have been selected in order to assure a well-defined first-order-transition behavior of the opinion formation model, as requested for the observation of DPT's in physical systems.

A. Dynamic response of the social group to changes in the opinion of the leader

We analyzed this situation in the absence of the influence of the mass media ($h \equiv 0$), so that the results are also independent of β. For this case one has the advantage that there is not a preferred state of opinion, and furthermore, one can evaluate the influence of the leader separately. In order to get insight into the relaxation times characteristic of the system, we first set the opinion of the leader constant $\sigma_L \equiv 1$, while the remaining $N-1$ individuals have the opposite opinion $\sigma_i = -1$ at time $t=0$. Figure 1 shows the response of the social group when it is measured for different values of the strength

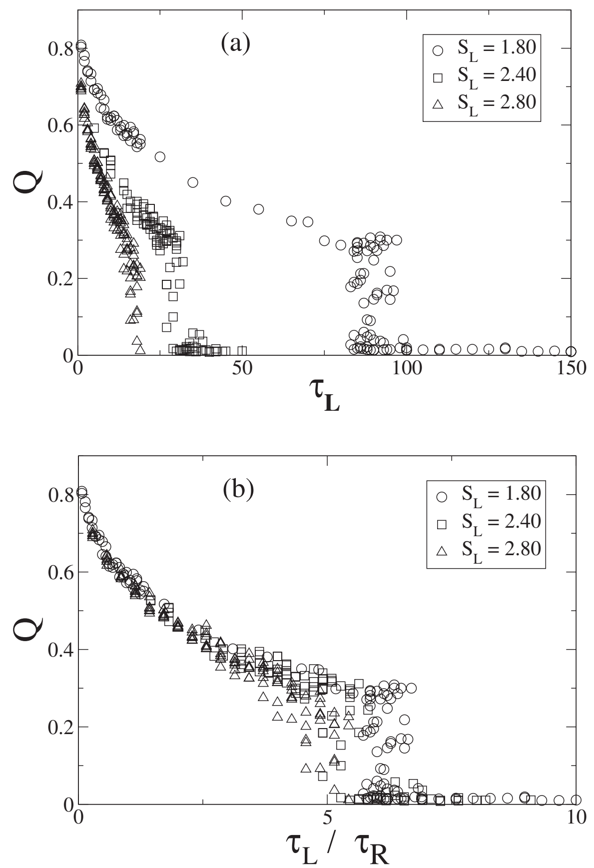

FIG. 2. (a) Plot of the dynamic order parameter Q versus the period τ_L of the external drive given by the change in the opinion of the leader. Results corresponding to different values of the strength of the leader S_L are shown. Data obtained for $N=2048$ individuals. (b) Data already shown in (a) but plotted versus the period rescaled by the relaxation time (τ_L/τ_R). More details in the text.

of the leader S_L. As follows from Fig. 1, one observes that longer relaxation times are necessary for weaker leaders, as expected. Then we define the relaxation time τ_R, as the time required to reach a balanced opinion $\sigma(\tau_R)\equiv 0$. The dependence of τ_R on S_L is shown in the inset of Fig. 1. Now, in order to study the dynamic behavior of the social group under the influence of the leader, we assumed that his/her opinion changes from $\sigma_L=1$ to $\sigma_L=-1$ according to a square-wave function of period τ_L and, of course, the strength S_L remains constant while the mass media are switched off ($h=0$). Figure 2(a) shows plots of the dynamic order parameter Q versus τ_L as obtained for different values of S_L. By analyzing the behavior of the data corresponding to $S_L=1.8$ one concludes that for short periods ($\tau_L<90$) the system can no longer follow the changes of the opinion of the leader and consequently one has a nonvanishing dynamic order parameter ($Q>0$). On the other hand, for $\tau_L>90$ the system gently follows the leader giving $Q\simeq 0$. For intermediate values of τ_L one observes an abrupt drop in Q, from $Q\simeq 0.30$ to $Q\simeq 0.01$, which takes place close to $\tau_L^{\mathrm{coex}}\simeq 90$. This result, as well as the observation of metastabilities close to τ_L^{coex} and within the interval $81<\tau_L^{\mathrm{coex}}<99$, strongly suggests a first-order transitionlike behavior. Of course, this abrupt change of Q may be considered as the precursor of a true far-from-equilibrium first-order DPT that would be observed in the thermodynamic limit ($N\to\infty$) only. While the curve corresponding to $S_L=2.4$ also shows an abrupt drop in Q at a certain "coexistence" value of the period ($\tau_L^{\mathrm{coex}}\simeq 29$), the behavior of the system for other values of S_L, say for $S_L\leqslant 1.8$, is not so clear, and the order of the "transition" is no longer easy to assign. However, by rescaling the horizontal axis of Fig. 2(a) by the relaxation time τ_R, as already obtained by using the data shown in the inset of Fig. 1, one observes an acceptable data collapse of the results corresponding to different values of S_L, as shown in Fig. 2(b). This way of plotting the data provides evidence showing that for all the studied values of S_L there are abrupt changes of the dynamic order parameter close to a given (normalized) coexistence point, which by proper rescaling is found to be of the order of $\tau_L/\tau_R\simeq 6$. Of course, the metastabilities already observed in the raw data shown in Fig. 2(a) are also observed in the rescaled plot of Fig. 2(b). Few results obtained for $N=4096$ individuals show negligible finite-size effects, as expected for first-order tranitions, strongly suggesting that the observed behavior would also hold for larger samples. Due to this observation and the fact that simulations in large systems are quite CPU-time demanding due to the presence of metastablitities, we have not performed a detailed finite-size scaling analysis.

B. Dynamic response of the social group driven by the mass media

In order to study this situation, we assume that, in contrast to the previously studied case, the opinion of the leader remains unchanged during all the simulation ($\sigma_L\equiv\sigma_1=1$), while the mass media change their states from $h=h_0>0$ to $h=-h_0<0$ with a period τ_{MM}. So, h_0 is the "amplitude" of the square wave function that describes the oscillation of the mass media. Of course, the presence of the leader with a well-defined state of opinion breaks the symmetry of the system, so one would expect a departure from the standard behavior observed by studying DPT's when the system is driven between two symmetric states, as in the case studied in the previous subsection and, e.g., in the typical Ising model.

Figure 3 shows plots of Q versus τ_{MM}, as obtained for different values of the amplitude h_0 of the mass media. These results were obtained by taking $S_L=2.8$ for the strength of the leader, but we found that the overall behavior remains qualitatively unchanged for $1.8<S_L<3.0$. All data shown in Fig. 3 is consistent with relatively large values of Q for short periods, as expected since the social group may not be able to follow a high-frequency external drive imposed by the mass media. On the other hand, for longer periods one observes that Q always reaches a saturation plateau (Q_s) that depends on the amplitude of the drive. Furthermore, one has that $Q_s\to 0$ when h_0 increases (see Fig. 3). In view of these results, we plotted Q_s versus h_0, as shown in the inset of Fig. 3. Here one observes that Q_s drops abruptly and beyond a certain critical amplitude $h_0^c\simeq 0.02$ the saturation value of the dynamic order parameter is negligible ($Q_s\to 0$), indicating that the system has reached a dynamically disordered state.

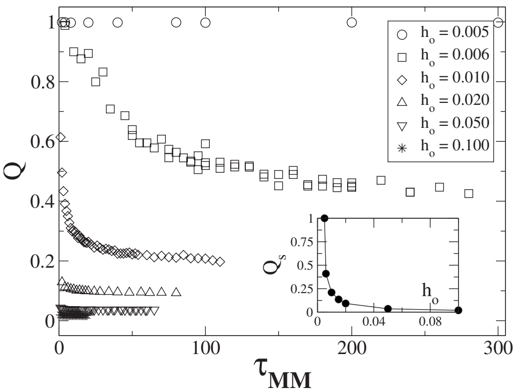

FIG. 3. Plots of the dynamic order parameter Q versus the period τ_{MM} of the external drive given by the change in the opinion of the mass media when the opinion of the leader remains unchanged at $\sigma_L=1$. Data obtained for different amplitudes (h_0) of the mass media as indicated. The inset shows the dependence of the saturation value of the dynamic order parameter Q_s as a function of the amplitude of the external drive h_0. Data obtained for $N=2048$ individuals. More details in the text.

Within this state the social group tends to follow the external drive. The fact that one requires a critical amplitude threshold for the observation of this regime is consistent with the symmetry breaking imposed by the leader that stands for a fixed opinion state.

V. CONCLUSIONS

We studied the dynamic response of a social group upon periodic variations in the state of opinion of a strong leader, as well as upon the variations of the influence of the mass media when the leader stands for a defined opinion state.

The influence of a leader changing his/her opinion periodically causes the occurrence of DPT's between a dynamically ordered state when the social group is not able to follow the leader (high-frequency regime), and a dynamically ordered state when the group is driven by the leader (low-frequency regime). The observed DPT's are abrupt and resemble a first-order-like behavior.

On the other hand, when the opinion of the mass media is changed periodically but the leader's opinion remains fixed, it is found that one requires an amplitude threshold in order to achieve a dynamically disordered regime with the social group essentially driven by the external input. This threshold is due to the fact that one has to overcome the influence of the leader, a situation that can only be achieved for a large enough amplitude value of the mass media.

It is worth mentioning that the dynamic behavior of the studied opinion formation model shares many features with that observed in archetypical interacting particle systems studied in the fields of condensed matter and statistical physics, such as magnetic systems, catalyzed reactions, and the underpotential deposition of metals.

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